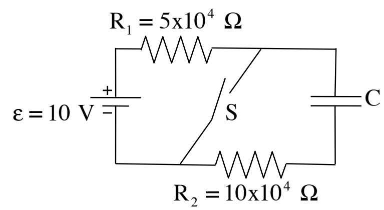
Problem 28.39



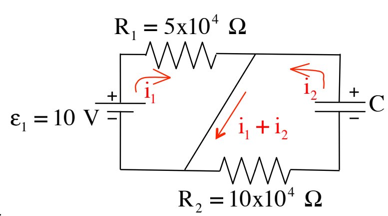
where $C = 10x10^{-6}\ \Omega$

a.) With the switch open a long time, the capacitor will be fully charged and there will be no current in the circuit. While the capacitor is charging, though, the time constant will be:

$$\begin{aligned}\tau_{charging} &= R_{net}C \\ &= [\ R_1 + R_2\ ](C) \\ &= \left[\left(5x10^4\ \Omega\right)+\left(10x10^4\ \Omega\right)\right]\left(10x10^{-6}\ f\right) \\ &= 1.50 \text{ seconds}\end{aligned}$$

b.) When the switch is thrown and the capacitor begins to discharge through R_2, the time constant will be:

$$\begin{aligned}\tau_{charging} &= R_2C \\ &= \left(10x10^6\ \Omega\right)\left(10x10^{-6}\ f\right) \\ &= 1 \text{ seconds}\end{aligned}$$



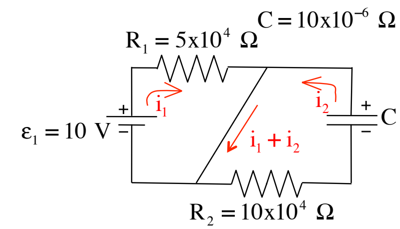
where $C = 10x10^{-6}\ \Omega$

c.) This is where things get interesting. As was said above, with the switch opened for a long time, the current goes to zero as the capacitor charges to its maximum, and the voltage across the capacitor at that point is $\varepsilon = 10\ V$.

When the switch it thrown, then, a current will be generated through R_1 due to the battery and a current will be generated through R_2 due to the voltage across the capacitor. All of these currents are shown in the sketch.

From Ohm's Law on the left loop, we can write:

$$\begin{aligned}&\varepsilon_1 - i_1R_1 = 0 \\ \Rightarrow\ & i_1 = \frac{\varepsilon_1}{R_1} \\ \Rightarrow\ & i_1 = \frac{(10\ V)}{\left(5x10^4\ \Omega\right)} \\ \Rightarrow\ & \quad = 2x10^{-4}\ A\end{aligned}$$



where $C = 10x10^{-6}\ \Omega$

Remembering that $V_{cap} = \frac{q(t)}{C}$, Ohm's Law used on the right loop yields:

$$\begin{aligned}&\frac{q(t)}{C} - i_2R_2 = 0 \\ \Rightarrow\ & i_2 = \left(\frac{1}{R_2C}\right)q(t)\end{aligned}$$

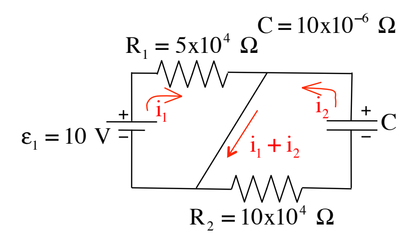
We know from the definition of capacitance that:

$$C = Q_{max}/V_{max} \quad \Rightarrow \quad Q_{max} = CV_{max}$$

and we know from previous experience that the charge on a discharging capacitor is equal to: $q(t) = Q_{max}e^{-t/R_2C}$

Coupling those two bits of information allows us to write:

$$\begin{aligned}q(t) &= Q_{max}e^{-t/R_2C} \\ &= CV_{max}e^{-t/R_2C}\end{aligned}$$



where $C = 10x10^{-6}\ \Omega$

With that, we can write:

$$\begin{aligned}i_2 &= \left(\frac{1}{R_2C}\right)q(t) \\ &= \left(\frac{1}{R_2C}\right)\left(CV_{max}e^{-t/R_2C}\right) \\ &= \left(\frac{1}{R_2}\right)\left(V_{max}e^{-t/R_2C}\right) \\ &= \left(\frac{1}{10x10^4\ \Omega}\right)\left((10\ V)e^{-t/\left(10x10^4\right)\left(10x10^{-6}\right)}\right) \\ &= \left(10^{-4}\right)e^{-t/(1\ sec)}\end{aligned}$$

That means the switch carries a downward current of:

$$i_1 + i_2 = \left(2x10^{-4}\ A\right) + \left[\left(1x10^{-4}\right)e^{-t/(1\ sec)}\ A\right]$$

Crazy, huh?